The axial anomaly and the conversion of gluons into photons [1]

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Abstract

The axial anomaly in the divergence of the singlet axial current in QCD + QED leads to low-energy theorems for the matrix element of this operator equation over vacuum and two–photon states and for the matrix element over vacuum and two–gluon states. The solution of these theorems is related only to the nonperturbative phenomena. These matrix elements are calculated in instanton vacuum generated N-JL type quark model for arbitrary N_f. It is shown that this model does satisfy the low-energy theorems.

1. Introduction.

The axial anomaly leads to many interesting nonperturbative phenomena in physics. Among them are B-violation processes in electroweak (EW) physics, $U_A(1)$ problem in QCD etc. The solution of these problems is intimately related to the topologically nontrivial structure of the vacuum in the gauge theories.

In this paper we apply the axial anomaly low-energy theorems [1] to test the chiral quark model [2] which is based on the instanton model of QCD vacuum [3]. We find that this model does satisfy these theorems. This conclusion provides solid background to calculate the different amplitudes of nonperturbative conversion of gluons into hadrons and photons.

2. The low-energy theorems and axial anomaly.

The axial anomaly in the divergence of the singlet axial current in QCD + QED leads to a low-energy theorem for the matrix elements of this operator equation over vacuum and two–photon states:

$$\langle 0|N_f \frac{g^2}{32\pi^2} G\tilde{G}|2\gamma\rangle = N_c \frac{e^2}{8\pi^2} \sum_f Q_f^2 F^{(1)}\tilde{F}^{(2)} \tag{1}$$

at $q^2 = 0$ [1]. Here, N_f is the number of the flavors, g is QCD coupling constant with $2G\tilde{G} = \epsilon^{\mu\nu\lambda\sigma}G_{\mu\nu}^a G_{\lambda\sigma}^a$ and $G_{\mu\nu}^a$ being the operator of the gluon field strengths , N_c is the number of the colors, e and Q_f are QED coupling constant and the electric charges

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of the quarks, respectively, $2F^{(1)}\tilde{F}^{(2)} = \epsilon^{\mu\nu\lambda\sigma}F_{\mu\nu}^{(1)}F_{\lambda\sigma}^{(2)}$, $F_{\mu\nu}^{(i)} = \epsilon_{\mu}^{(i)}q_{i\nu} - \epsilon_{\nu}^{(i)}q_{i\mu}$, $\epsilon_{\mu}^{(1,2)}$, $q_{1,2}$ are polarizations and momenta of photons respectively and $q = q_1 + q_2$. This relation is a consequence of the absence of a massless singlet pseudoscalar boson. Here the contribution of the quark masses is neglected.

So, the problem is reduced to the calculation of the matrix element:

$$\langle 0|g^2 G\tilde{G}|2\gamma\rangle = \epsilon_{\mu}^{(1)}\epsilon_{\nu}^{(2)}\int \langle 0|T(g^2 G\tilde{G} j_{\mu}^{em}(x_1)j_{\nu}^{em}(x_2)|0\rangle \exp i(q_1 x_1 + q_2 x_2)dx_1 dx_2. \quad (2)$$

As it is evident, gluons can interact with photons only through quark loops. In perturbation theory it leads to at least $\sim g^4$ result for the left hand side of Eq. (1) (see e.g. recent discussion of the higher-loop contributions to the axial anomaly [4]). So, the solution of this theorem is related only to the nonperturbative phenomena connected with the structure of QCD vacuum.

Another nontrivial low-energy theorem concerns the matrix element over vacuum and two-gluon states:

$$\langle 0|g^2 G\tilde{G}|2gluons\rangle = 0 \quad (3)$$

at the limit $q^2 = 0$.

These matrix elements are calculated in the instanton vacuum generated N-JL type quark model [2], [5] for arbitrary N_f.

3. The instanton vacuum of QCD.

The instanton is the solution of gluodynamics in the Euclidian space[6]:

$$A_{\mu}^{Ia}(x) = 2g^{-1}O_I^{ab}\bar{\eta}_{\mu\alpha}^b \frac{\rho^2(x-z)_{\nu}}{[(x-z)^2 + \rho^2](x-z)^2}, \quad (4)$$

and in the momentum representation and at small k^2 it is:

$$A_{\mu}^{Ia}(k) = O_I^{ab}\bar{\eta}_{\mu\alpha}^b \frac{i4\pi^2\rho^2 k_{\nu}}{gk^2}(1 + O(k^2\rho^2)). \quad (5)$$

The anti-instanton solution has the same form as in Eqs. (4) and (5) but with $\eta_{\mu\alpha}^b$ instead of $\bar{\eta}_{\mu\alpha}^b$. Here $O_{I(\bar{I})}$ is the orientation matrix of the instanton I (anti-instanton $\bar{I}$) in color space, $\bar{\eta}(\eta)$ – t'Hooft factors [7], ρ is the size and z is the position of the instanton. For large interinstanton distances $R >> \rho$ the sum of the instantons and the anti-instantons is also an approximate solution. The calculation of the action for gluon fields leads to sum of the actions of free instantons and classical interinstanton potential $V(R, \rho_1, \rho_2, O)$, where $O = O_1^T O_2$ is a matrix of relative orientation. The most important part is the instanton-antiinstanton potential $V_{I\bar{I}}$. As it is well–known, at large distances it has a form

$$V_{I\bar{I}} = \frac{32\pi^2}{g^2}\frac{D\rho_1^2\rho_2^2}{R^4} \quad (6)$$

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where orientation factor D is

$$D = \bar{\eta}^a_{\mu\alpha} O^{ab} \eta^b_{\mu\beta} \left(\frac{R_\alpha R_\beta}{R^2} - \frac{1}{4} g_{\alpha\beta} \right). \tag{7}$$

This resembles the dipole-dipole interaction potential and may be attractive.

The main assumption of the model concerns small distances $R \sim \rho$. At small distances it is assumed that there is repulsion. This assumption is supported by both phenomenological and theoretical consideration [3], [8] (see recent discussion of this "Interacting Instanton Liquid Model" in [9] and references therein). This leads to a stabilization of the size and density of instantons. The distribution of the number of pseudoparticles should, for large $\langle N \rangle$, be given by

$$P(N) \propto \exp\left[-\frac{b}{4} N \left(\log \frac{N}{\langle N \rangle} - 1 \right) \right], \qquad N_- = N_+ = N/2 \tag{8}$$

and the effective size distribution of I's and $\bar{I}$'s coincides, and becomes

$$d(\rho) = \text{const} \times \rho^{b-5} \exp\left[-\frac{b-4}{2} \frac{\rho^2}{\overline{\overline{\rho^2}}} \right]. \tag{9}$$

When fermions are included (N_f flavors), the coefficient of the beta function b is

$$b = \frac{11}{3} N_c - \frac{2}{3} N_f. \tag{10}$$

The quantities $\langle N \rangle$ and $\bar{\rho}$ include effects of the instantons interactions [5]. In the following calculations, for simplicity, all instanton sizes are considered to be $\bar{\rho}$.

Both the phenomenological estimates and variational calculations (see recent discussion in [9]) lead to a mean interinstanton distance of $\bar{R} = \left(\frac{V}{\langle N \rangle} \right)^{1/4} \sim 1\, fm$ and a mean instanton size of $\bar{\rho} \sim 1/3\, fm$. The small packing parameter $(\bar{\rho}/\bar{R})^4 = 0.012$ provides a possibility for the independent averaging over positions and orientations of instantons.

4. The chiral quark model.

The main assumption of this model is the interpolation formula for the quark propagator in the single instanton field. It is approximated as the sum of a free propagator and an explicit contribution of the zero mode [2],

$$\left(i\hat{\nabla}(\xi_{I(\bar{I})}) + im \right)^{-1}_{1-\text{inst}} \approx (i\hat{\partial})^{-1} - \frac{\Phi_\pm(x; \xi_{I(\bar{I})}) \Phi^\dagger_\pm(y; \xi_{I(\bar{I})})}{im}. \tag{11}$$

Here, $\hat{\nabla} == \hat{\partial} - ig\hat{A}$, $\Phi_\pm(x; \xi_{I(\bar{I})})$ is the zero mode wave function of the fermion in the background of one $I(\bar{I})$[7]. It depends on the collective instanton variables $\xi_{I(\bar{I})}$ – the size ρ, the position z and the orientation O of the instanton.

This interpolating formula should be accurate both at small momenta $(p \ll 1/\bar{\rho})$, where the zero mode is dominant, and at large momenta $(p \gg 1/\bar{\rho})$, where the propagator reduces to the free one. In the background of an $N_\pm$–instanton configuration and keeping in mind the low density of the instanton media this formula leads to the partition function of the model:

$$Z_N = \int D\psi D\psi^\dagger \exp(\int d^4x \, \psi^\dagger i\hat{\partial}\psi) \, W_+^{N_+} \, W_-^{N_-}, \tag{12}$$

where

$$W_\pm = \left(-\frac{4\pi^2\bar{\rho}^2}{N_c}\right)^{N_f} \int \frac{d^4z}{V} \det J_\pm(z), \tag{13}$$

$$J_\pm(z)_{fg} = \int \frac{d^4k d^4l}{(2\pi)^8} \exp(-i(k-l)z) \, F(k)F(l) \, \psi_f^\dagger(k)\tfrac{1}{2}(1 \pm \gamma_5)\psi_g(l), \tag{14}$$

and the contribution of the current quark masses is neglected. The form-factor F is related to the zero–mode wave function in momentum space $\Phi_\pm(k; \xi_{I(\bar{I})})$ and is equal to:

$$F(k) = -t\frac{d}{dt}\left[I_0(t)K_0(t) - I_1(t)K_1(t)\right] \rightarrow \begin{cases} 1 & t \rightarrow 0 \\ \frac{3}{4}t^{-3} & t \rightarrow \infty \end{cases}, \tag{15}$$

with $t = \frac{1}{2}k\bar{\rho}$.

The formula:

$$(ab)^N = \int d\lambda \exp(Nln\frac{aN}{\lambda} - N + \lambda b). \tag{16}$$

(here $N >> 1$) provides the final expression for the partition function [5]:

$$Z_N = \int D\psi D\psi^\dagger \exp(-S_{eff}), \tag{17}$$

where

$$-S_{eff} = \int \psi^\dagger i\hat{\partial}\psi + Y_+ + Y_-, \tag{18}$$

and

$$Y_\pm = (i)^{N_f}\lambda \int d^4z \, \det J_\pm(z) = \left(\frac{2V}{N}\right)^{N_f-1} (iM)^{N_f} \int d^4z \, \det J_\pm(z). \tag{19}$$

The self-consistency condition at the saddle point in Eqs. (17), (18) and (19) leads to

$$4N_cV \int \frac{d^4k}{(2\pi)^4}\frac{M^2F^4(k)}{M^2F^4(k) + k^2} = N. \tag{20}$$

5. Calculations with the low-energy theorems.

In the quasiclassical(saddle-point) approximation any gluon operator receives its main contribution from the instanton background. As an example, for one instanton (anti-instanton)$I(\bar{I})$,

$$g^2 G^2(x) = \frac{192\rho^4}{[\rho^2 + (x-z)^2]^4} = f(x-z), \tag{21}$$

and

$$g^2 G\tilde{G}(x) = \pm f(x-z). \tag{22}$$

In the following the operator $g^2 G\tilde{G}(x)$ is considered. Given the low density of the instanton medium it is possible to neglect the overlap of the fields of different instantons. In that case, the matrix element of the gluon operator $G\tilde{G}(x)$ with any other quark operator Q is:

$$\langle g^2 G\tilde{G}(x)Q\rangle_N = Z_N^{-1} \int D\psi D\psi^\dagger \exp(\int \psi^\dagger i\partial\psi)$$

$$\times \left(N_+ \left(W_{G\tilde{G}+}(x)Q\right) W_+^{N_+-1} W_-^{N_-} + N_- \left(W_{G\tilde{G}-}(x)Q\right) W_+^{N_+} W_-^{N_--1}\right), \tag{23}$$

where

$$W_{G\tilde{G}\pm} = \pm \left(-\frac{4\pi^2 \bar{\rho}^2}{N_c}\right)^{N_f} \int d^4z \, f(x-z) \det J_\pm(z). \tag{24}$$

The application of the formula (16) leads to

$$\langle g^2 G\tilde{G}(x)Q\rangle_N = Z_N^{-1} \int D\psi D\psi^\dagger \exp(-S_{eff}) \left(\left(Y_{G\tilde{G}+}(x) + Y_{G\tilde{G}-}(x)\right) Q\right), \tag{25}$$

where

$$Y_{G\tilde{G}\pm}(x) = \pm \left(\frac{2V}{N}\right)^{N_f-1} (iM)^{N_f} \int d^4z \, f(x-z) \det J_\pm(z), \tag{26}$$

and we take $N_\pm = N/2$ to preserve CP invariance.

In this particular case, the calculation of the matrix element, Eq. (2), can be reduced to the calculation of the the partition function

$$\hat{Z}_N[\kappa, a] = Z_N^{-1} \int D\psi D\psi^\dagger \exp(-\hat{S}_{eff}), \tag{27}$$

with the effective action, $\hat{S}_{eff}$, in the presence of an external electromagnetic field, a_μ, and an external field, $\kappa(x)$, is given as

$$-\hat{S}_{eff} = \int \psi^\dagger i\hat{D}\psi + Y_+ + Y_- + \int dx \left(Y_{G\tilde{G}+}(x) + Y_{G\tilde{G}-}(x)\right) \kappa(x), \tag{28}$$

where $\hat{D} = \hat{\partial} - ieQ_f \hat{a}$.

It is clear that

$$\langle 0|T(g^2 G\tilde{G}(x) j_\mu^{em}(x_1) j_\nu^{em}(x_2)|0\rangle = \frac{\delta \hat{Z}_N[\kappa, a]}{\delta\kappa(x)\delta a_\mu(x_1)\delta a_\nu(x_2)}\Big|_{\kappa,a=0} \tag{29}$$

Finally, Eq. (28) can be rewritten as:

$$
\begin{aligned}
-\hat{S}_{eff} = &\int \psi^\dagger i \hat{D} \psi + \\
&\left(\tfrac{2V}{N}\right)^{N_f-1} \int dz \ (1 + \int dx \kappa(x) f(x-z)) \det(iMJ_+(z)) + \\
&\left(\tfrac{2V}{N}\right)^{N_f-1} \int dz \ (1 - \int dx \kappa(x) f(x-z)) \det(iMJ_-(z)).
\end{aligned}
\tag{30}
$$

6. Bosonization of the partition function $\hat{Z}_N[\kappa, a]$.

Another remarkable formula

$$
\exp(\lambda \det[iA]) = \int d\mathcal{M} \exp\left[-(N_f-1)\lambda^{-\frac{1}{N_f-1}}(\det \mathcal{M})^{\frac{1}{N_f-1}} + itr(\mathcal{M}A)\right]
\tag{31}
$$

is used in the following discussion. It is possible to check this by the saddle point approximation of the integral.

It is convenient to introduce:

$$
M_\pm(z) = \left(1 \pm \int dx \kappa(x) f(x-z)\right)^{(N_f-1)^{-1}} M
$$

and rewrite

$$
\begin{aligned}
-\hat{S}_{eff} = &\int \psi^\dagger i \hat{D} \psi + \\
&\left(\tfrac{2V}{N}\right)^{N_f-1} \int dz \ \det(iM_+(z)J_+(z)) + \left(\tfrac{2V}{N}\right)^{N_f-1} \int dz \ \det(iM_-(z)J_-(z)).
\end{aligned}
\tag{32}
$$

By using the formula, Eq.(31), it is easy to show that:

$$
\begin{aligned}
&\exp\left(\left(\tfrac{2V}{N}\right)^{N_f-1} \int dz \ \det(iM_\pm(z)J_\pm(z))\right) = \\
&\int D\mathcal{M}_\pm \exp\left[\int dz \left(-(N_f-1)\left(\tfrac{2V}{N}\right)^{-1}(\det \mathcal{M}_\pm)^{\frac{1}{N_f-1}}\right) + iTr(\mathcal{M}_\pm M_\pm J_\pm)\right]
\end{aligned}
\tag{33}
$$

By using this relation, Eq.(33), the path integral over quarks in the partition function, Eq.(27), the effective action $\hat{S}_{eff}$, describing mesons in the presence of the external fields a_μ and κ is

$$
-\hat{S}_{eff}[\mathcal{M}_\pm, a, \kappa] = \int dz \left(-(N_f-1)\left(\tfrac{2V}{N}\right)^{-1}(\det \mathcal{M}_\pm)^{\frac{1}{N_f-1}}\right) +
$$

$$
Tr \ln\left(i\hat{D} + i\mathcal{M}_+ MF^2 (1+(\kappa f))^{N_f^{-1}} \tfrac{1}{2}(1+\gamma_5) + i\mathcal{M}_- MF^2 (1-(\kappa f))^{N_f^{-1}} \tfrac{1}{2}(1-\gamma_5)\right)
\tag{34}
$$

For the processes without mesons the partition function is:

$$\hat{Z}_N[\kappa, a] =$$

$$\exp Tr \ln \left(i\hat{D} + iMF^2 \left(1 + (\kappa f)\right)^{N_f^{-1}} \tfrac{1}{2}(1 + \gamma_5) + iMF^2 \left(1 - (\kappa f)\right)^{N_f^{-1}} \tfrac{1}{2}(1 - \gamma_5) \right)$$

$$\times \left(i\hat{\partial} + iMF^2 \right)^{-1},$$

(35)

where $(\kappa f) = \int dx \kappa(x) f(x - z)$.

7. The low-energy theorem for the matrix element between vacuum and two-photons states.

The matrix element, Eq. (2), is generated by

$$\frac{\delta \hat{Z}_N[\kappa, a]}{\delta \kappa(x)\delta a_\mu(x_1)\delta a_\nu(x_2)}\Big|_{\kappa, a=0}$$
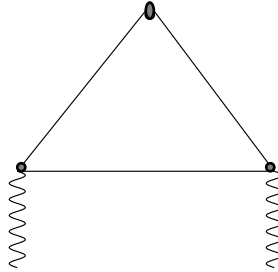
and is given by the Feynman diagram, Fig.1:

Fig. 1

As it is clear from Eq. (35) the factors in the vertices in the diagram are $eQ_f\gamma_\mu$ and $iMfF^2\gamma_5 N_f^{-1}$.

We must calculate $\Delta(q^2)$ (in the $\lim q^2 \to 0$), which is defined by:

$$(2\pi)^4\delta(q - q_1 - q_2)\Delta(q^2) = \int dx \exp(-iqx)\langle 0|g^2 G\tilde{G}(x)|2\gamma\rangle =$$

$$e^2\epsilon_\mu^{(1)}\epsilon_\nu^{(2)} \int \langle 0|T(g^2 G\tilde{G}(x)j_\mu^{em}(x_1)j_\nu^{em}(x_2)|0\rangle \exp i(-iqx + q_1 x_1 + q_2 x_2)dxdx_1 dx_2.$$

(36)

It is clear from the previous consideration that

$$\Delta(q^2) = \epsilon_\mu^{(1)}\epsilon_\nu^{(2)} f(q^2)N_c e^2 \sum_f Q_f^2$$

$$\times Tr[\int \frac{d^4 p}{(2\pi)^4} \frac{iMF_1 F_2\gamma_5(\hat{p} - \hat{q}_1 + iMF_1^2)\gamma_\mu(\hat{p} + iMF_3^2)\gamma_\nu(\hat{p} + \hat{q}_2 + iMF_2^2)}{((p - q_1)^2 + M^2 F_1^4)(p^2 + M^2 F_3^4)((p + q_2)^2 + M^2 F_2^4)}$$

$$+ (\mu \rightleftharpoons \nu, \; q_1 \rightleftharpoons q_2)].$$

(37)

Here $F_1 = F((p - q_1))$, $F_2 = F(p + q_2)$, $F_3 = F(p)$ and

$$f(q^2) = \int dx \exp(-iqx)f(x) \tag{38}$$

is the form-factor of the one-instanton contribution to $g^2 G\tilde{G}$. At $q^2 = 0$

$$f(q^2 = 0) = 32\pi^2.$$

It is easy to show that the trace in Eq.(37) can be reduced to

$$8M^2 \epsilon^{\mu\nu\lambda\sigma} q_{1\lambda} q_{2\sigma} \Gamma(q^2),$$

where

$$\Gamma(q^2) = \int \frac{d^4 p}{(2\pi^4)} \frac{F_1 F_2 F_3^2}{((p - q_1)^2 + M^2 F_1^4)(p^2 + M^2 F_3^4)((p + q_2)^2 + M^2 F_2^4)} + \dots \tag{39}$$

It is possible to calculate this integral if we put $F = 1$. In this approximation

$$\Gamma(q^2) = \frac{1}{16\pi^2 q^2} \int_0^1 \frac{dx}{1 - x} ln\left(1 + \frac{x(1 - x)q^2}{M^2}\right). \tag{40}$$

At a small values of q^2, we have

$$\Gamma(q^2) = \frac{1}{32\pi^2 M^2}\left(1 - \frac{q^2}{12M^2}\right) \tag{41}$$

As a result, the left side of the low-energy theorem, Eq.(1), is

$$(N_f \frac{g^2}{32\pi^2})(\frac{4e^2 N_c}{g^2 N_f} \sum_f Q_f^2)F^{(1)}\tilde{F}^{(2)} \tag{42}$$

and this coincides with the right side of Eq.(1).

8. The low-energy theorem for the matrix element between vacuum and two-gluons states.

Here we present, without details, calculations related to Eq. (3). This matrix element can be written in the form:

$$\langle 0|g^2 G\tilde{G}|g(\epsilon^{(1)}, q_1), g(\epsilon^{(2)}, q_2)\rangle = \epsilon_{\mu_1}^{(1)a_1} \epsilon_{\mu_2}^{(2)a_2}$$

$$\times \int \partial_2^2 \partial_1^2 \langle 0|Tg^2 G\tilde{G}A_{\mu_1}^{a_1}(x_1)A_{\mu_2}^{a_2}(x_2)|0\rangle \exp i(q_1 x_1 + q_2 x_2)dx_1 dx_2. \tag{43}$$

Here $A_\mu^a(x)$ is a total gluon field, $\epsilon_{\mu_i}^{(i)a_i}$, q_i are the polarization and the momentum of gluons respectively.

As usual, we expand the total field $A^a_\mu(x)$ around the instanton background. The main term in Eq.(43) is the contribution of the instanton background and is order $\sim O(g^{-2})$. The next term is the contribution of the perturbative fluctuations over instanton background and is order $\sim O(g^2)$. It is easy to see from previous considerations that $O(g^{-2})$ term is given by the formula

$$Z_N^{-1} \int D\psi D\psi^\dagger \exp(-S_{eff}) \left(\left(Y_{G\tilde{G}AA+}(x) + Y_{G\tilde{G}AA-}(x) \right) Q \right), \qquad (44)$$

where

$$Y_{G\tilde{G}AA\pm} = \pm \left(\frac{2V}{N} \right)^{N_f-1} (iM)^{N_f} \int d^4z\, f(x-z)$$
$$\times \int dO (-\partial_1^2) A^{I(\bar{I})a_1}_{\mu_1}(x_1)(-\partial_i^2) A^{I(\bar{I})a_2}_{\mu_2}(x_2) \det J_\pm(z), \qquad (45)$$

Here the instanton(anti-instanton) is located at the point z with its orientation O.

Repeating of the bosonization trick leads to the result for the $O(g^{-2})$ contribution which is proportional to

$$Tr[(i\hat{\partial} + iMF^2)^{-1} iMF^2 \gamma_5].$$

It is clear that this Tr and as a consequence $O(g^{-2})$ term are equal to zero.

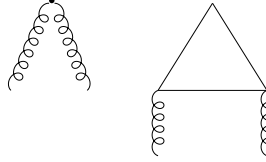The next $O(g^2)$ term is the contribution of the diagrams, Fig.2.

Fig. 2

It is clear that the first diagram is the direct contribution of the operator $g^2 G\tilde{G}$ which is equal to

$$-g^2 G^{(1)}\tilde{G}^{(2)},$$

where $2G^{(1)}\tilde{G}^{(2)} = \epsilon^{\mu\nu\lambda\sigma} G^{(1)a}_{\mu\nu} G^{(2)a}_{\lambda\sigma}$, $G^{(i)a}_{\mu\nu} = \epsilon^{(i)}_\mu q_{i\nu} - \epsilon^{(i)}_\nu q_{i\mu}$.

The factors in the vertices of the second loop–diagram are $g\lambda_a/2\gamma_\mu$ and $iMfF^2\gamma_5 N_f^{-1}$. An account of the contribution from all flavors gives the coefficient N_f.

A comparison with the previous calculations (Eq.(36), (37)) leads to the result that the contribution of the second loop–diagram is equal in magnitude to the contribution of the first diagram at $q^2 = 0$ but of opposite in sign.

Then, terms of $O(g^2)$ are equal to zero in the limit $q^2 \to 0$.

9. Conclusion.

Thus we conclude that the instanton vacuum generated chiral quark model satisfies the low-energy theorems, Eq.(1) and (3). This provides solid background to calculate the different amplitude of nonperturbative conversion of gluons into hadrons and photons.

We are planning to apply this approach to the transitions between the states in heavy quarks systems and to the $\gamma\gamma$ collision processes.

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